VOLUME 24 JUNE, 1938 NUMBER 6

DUKE UNIV. LIBRARY
DURHAM, N. C.
JUN 16 1938

PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

OF THE

UNITED STATES OF AMERICA

EXECUTIVE COMMITTEE

EDWIN BIDWELL WILSON, *Managing Editor*

RAYMOND PEARL LUDVIG HEKTOEN

PUBLISHED MONTHLY

Publication Office: Mack Printing Company, Easton, Pa.

Editorial Office: Harvard School of Public Health, Boston 17

Home Office of the Academy: Washington, D. C.

Entered as second-class matter June 21, 1920, at the post-office at Easton, Pennsylvania, under the act of August 24, 1912. Accepted for mailing at a special rate of postage, provided for in section 1103, act of October 3, 1917. Authorized June 18, 1920.

Price per annum, $5.00

OFFICERS OF THE ACADEMY

PRESIDENT ..... F. R. LILLIE
VICE PRESIDENT ..... A. L. DAY
FOREIGN SECRETARY ..... L. J. HENDERSON
HOME SECRETARY ..... FRED. E. WRIGHT
TREASURER ..... ARTHUR KEITH

EDITORIAL BOARD

The HOME SECRETARY and the FOREIGN SECRETARY of the ACADEMY
The CHAIRMAN OF THE NATIONAL RESEARCH COUNCIL

V. BUSH	L. J. HENDERSON	W. R. MILES
R. E. COKER	W. S. HUNTER	R. A. MILLIKAN
A. L. DAY	F. B. JEWETT	H. R. MOODY
L. P. EISENHART	A. B. LAMB	W. J. V. OSTERHOUT
R. G. HARRISON	E. R. LONG	F. K. RICHTMYER
L. HEKTOEN	C. R. LONGWELL	F. H. SEARES

INFORMATION TO SUBSCRIBERS

SUBSCRIPTIONS TO THE PROCEEDINGS OF THE NATIONAL ACADEMY OF SCIENCES at the rate of $5.00 per annum should be made payable to the National Academy of Sciences, and sent to the Home Secretary, National Academy of Sciences, Constitution Ave. & 21st Street, Washington, D. C. Single numbers, $0.50.

PAST publications of the NATIONAL ACADEMY OF SCIENCES from 1863 to 1926 and of the NATIONAL RESEARCH COUNCIL from 1916 to 1925 are listed in the Index to the First Ten Volumes of these PROCEEDINGS. This Index appeared as Part 2 of Volume 13, No. 1, January, 1927, and is for sale separately at the price of $1.50. Orders should be sent to the Home Secretary, National Academy of Sciences, Constitution Ave. & 21st Street, Washington, D. C.

PAST volumes of the PROCEEDINGS may be obtained at five dollars per volume unless the sale of the volume would break a complete set of Volumes I to X. Single numbers may be obtained for fifty cents except where the sale of such numbers would break up a complete volume. Only two hundred complete sets of the PROCEEDINGS are available for sale—Volume I to X, price $75.00 (including Index, see *supra*). Orders should be sent to the Home Secretary, National Academy of Sciences, Constitution Ave. & 21st Street, Washington, D. C.

The following publications are issued by the NATIONAL RESEARCH COUNCIL. Orders and inquiries should be addressed: Publication Office, National Research Council, Constitution Ave. & 21st Street, Washington, D. C.

(A) THE BULLETIN OF THE NATIONAL RESEARCH COUNCIL published at irregular intervals. Individual numbers priced variously. A discount of 20% from the list price will be given to all who subscribe for the series as issued, direct from the Publication Office.

(B) THE REPRINT AND CIRCULAR SERIES OF THE NATIONAL RESEARCH COUNCIL, individual numbers variously priced. A discount of 20% from the list price will be given to all who subscribe for the series as issued, direct from the Publication Office.

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
Volume 24 June 15, 1938 Number 6
Copyright 1938 by The National Academy of Sciences

EVIDENCE FOR A SECOND THIAMIN

By William J. Robbins and Frederick Kavanagh

The New York Botanical Garden

Communicated May 10, 1938

In an earlier paper[1] the authors reported briefly the effect on the growth of *Phycomyces Blakesleeanus* of each of 36 pyrimidines when used in a nutrient solution with 4-methyl-5β hydroxyethyl thiazole, one of the two intermediates from which Williams and Cline synthesized thiamin (vitamin B_1). Thirty of these compounds had been tested by the writers and six of them by other investigators. At that time we were unable to determine the significance of the CH_3 group in the second position on the pyrimidine ring because suitable compounds were not available.

Through the courtesy of the I. G. Farbenindustrie Aktiengesellschaft, samples of 5-bromo-methyl-6-amino pyrimidine and 2-ethyl-5-bromo-methyl-6-amino pyrimidine were secured. The first of these compounds differs from the pyrimidine used by Williams and Cline in having hydrogen substituted for the methyl group in the second position; the second compound has the ethyl group for the methyl group.

When ten units of 5-bromo-methyl-6-amino pyrimidine were used with ten units of the vitamin thiazole in the nutrient solution previously described[1] no growth of *Phycomyces* occurred. The second compound used in the same way was as effective as the 2-methyl-5-bromo-methyl-6-amino pyrimidine. For both sets of experiments the solutions containing the supplements were sterilized by heating at 15 lb. pressure for 15 minutes.

We conclude that for *Phycomyces* the methyl group in the second position on the vitamin pyrimidine is important since the substitution of hydrogen for the methyl group eliminated the activity of the pyrimidine. However, the effectiveness of the compound is largely or entirely retained when the ethyl group replaces the methyl group in the second position.

This observation suggests that there may be more than one compound capable of functioning in the living organism as thiamin does; a methyl thiamin, the compound originally isolated and synthesized, and an ethyl thiamin differing from the former in having the CH_3 group in the second position on the pyrimidine ring replaced by the C_2H_5 radical.

In our experiments we have used the pyrimidine and thiazole compounds as such. From evidence presented previously[2] we believe, however, that the vitamin molecule (not the intermediates themselves) functions in facilitating the growth of *Phycomyces*. When a mixture of a particular pyrimidine and a particular thiazole is effective we conclude that the vitamin molecule or a substitute therefor is synthesized by the organism from the intermediates. It is on this basis that we suggest the existence of an ethyl thiamin.

There is of course the possibility that the fungus was able to substitute the methyl for the ethyl group and that the final effective product was identical with thiamin. We have no evidence on this point, though we are inclined to assume that this is not probable.

If there is an ethyl thiamin as we suggest there may also be other thiamins such as propyl and butyl. The investigation of this possibility must necessarily wait on the availability of suitable pyrimidines.

[1] William J. Robbins and Frederick Kavanagh, *Proc. Nat. Acad. Sci.*, **24**, 141–145 (1938).

[2] William J. Robbins and Frederick Kavanagh, *Ibid.*, **23**, 499–502 (1937).

AN ASTACENE-LIKE CAROTENOID FROM A PACIFIC COAST ANEMONE, EPIACTIS PROLIFERA

BY DENIS L. FOX AND CHESNEY R. MOE

SCRIPPS INSTITUTION OF OCEANOGRAPHY, UNIVERSITY OF CALIFORNIA,* AND SAN DIEGO STATE COLLEGE

Communicated May 14, 1938

While many fishes, mollusks, crustaceans, anemones and members of other phyla contain known carotenoids of recognized plant origin, including any or several carotenes and xanthophylls, contributions from various investigators during recent years have brought out the fact that numerous marine animals contain also previously undescribed carotenoids as well, resulting in a steady extension of our list of known lipochromes peculiar to animals. These animal lipochromes fall chiefly into the classes of acidic and xanthophyllic carotenoids. (See reference in Zechmeister, 1934, 1937; Lederer, 1933, 1935, 1938; Heilbron, Jackson and Jones, 1935.)

Heilbron and his co-workers (op. cit.) have given some attention to lipochromes of several species of sea anemones, and report, among other pigments, including new animal xanthophylls, a deep red, acidic carotenoid ester from *Actinoloba dianthus*, which, when hydrolyzed and crystallized in pure condition, showed a single absorption maximum in CS_2 at 495 mμ.

From *Tealia felina* they obtained, along with other carotenoids, a different orange-red acid which showed a single absorption maximum in the same solvent of 500 mμ, whether free or in the natural esterified condition. While both of these pigments resembled astacene in their general color, acidic character and the possession of a single absorption band, neither was identical with it, since the positions of the respective bands were somewhat removed toward the violet from that of astacene, which is reported by Lederer (1935) to have an absorption maximum at 510 mμ in CS_2; Karrer and Solmssen (1935) report 514 mμ (497 mμ before hydrolysis) as the maximum of astacene in CS_2; they obtained the pigment from the sponges *Axinella crista-galli* and *Suberites domuncula*, as well as from a gastropod mollusk *Pleurobranchus elegans*, and a sea-star *Echinaster sepositus*. Additional critical data which set Heilbron's red carotenoids apart from astacene were in the melting points: while astacene melts at 240–243°C. (Zechmeister, 1934; Lederer, 1935), Heilbron's astacene-like carotenoid acids melted at 195–196°C. and 205–208°C., respectively.

Astacene is a tetraketo-beta-carotene (Karrer and Loewe, 1934). In view of the exceptional change brought about by various animals in an otherwise intact carotene or xanthophyllic molecule, resulting in the presence of four ketone groups (two on either terminal ionone ring) it would seem not unlikely that other typically animal carotenoids of similar but not identical physical and chemical properties might be synthesized by members of certain phyla. Of this we have already considerable evidence, as brought out above; facts reported below would appear to add information of similar significance.

An important question regarding the unique oxidative changes, such as are involved in the formation of astacene in animals is: Do those lipochromes which are peculiar only to animals result from biochemical differences in various animal species, all of which may modify a *common* carotenoid contained in their diet, or may the various animal species instead use each a *different* carotenoid as substrate for the synthesis? As an instance, we may consider for a moment the fact that taraxanthin, which seems to occur commonly in numerous marine forms (Fox, 1936; Young and Fox, 1936; see also citations in Zechmeister, 1937) has the empirical formula $C_{40}H_{56}O_4$ (Kuhn and Lederer, 1931), while the empirical formula for astacene is $C_{40}H_{48}O_4$—a difference of 8 H atoms, corresponding to the conversion of four secondary alcoholic groups to four ketonic groups. This particular picture is offered not as a suggestion of the exact compounds involved, but as a general working hypothesis for possible use as a guide in future metabolic studies to be made in pursuit of the problem.

This contribution concerns an interesting anemone, *Epiactis prolifera*, Verrill, found along the California coast from Puget Sound to San Diego (Johnson and Snook, 1927). Its young complete their development in

numerous circular pits around the outside of the parent's body, migrating later to positions on the substrate in the immediate vicinity. The color of this animal is red, orange-red or occasionally red-brown. It is of relatively small size, adults having a pedal disc diameter of 10–20 mm., and a similar height.

During June and July, 1936, great numbers of this species bearing young were taken from the kelp beds off the coast at San Diego, where they were found attached to the blades and stipes of the giant kelp *Macrocystis pyrifera*, along with other small animals, including species of the sedentary, tube-building serpulid worm *Spirorbis* and the small amphipod *Caprella*. We were led to believe that the anemone fed, at least in part, upon the latter amphipod.

When a few of the whole animals were ground in a mortar with sand and ethyl alcohol, the tissues yielded a yellow-orange extract. The epiphasic character of the pigment at this stage was demonstrated by its quantitative extraction from >90% alcohol by shaking with ligroin. Hydrolysis was brought about by treatment with alcoholic KOH for two or three hours in a hot water bath. Subsequently all pigment remained dissolved in the aqueous alcohol layer. Attempted extraction with ligroin now resulted in little or no yield of pigment to the latter solvent, even after diluting the alcohol layer with much water. Upon adding concentrated sodium chloride solution, the emulsion broke, the pigment being thrown out at the interface of the two liquids as a red potassium salt. Slight acidification of the system with acetic acid immediately hydrolyzed the potassium salt, and the free pigment was now dissolved in the ligroin layer. The neutral pigment was subjected to partition between 90% CH_3OH and ligroin; part of it migrated into the alcoholic layer below; on adding a drop of concentrated alkali all pigment was transferred from the ligroin phase into the alcoholic layer; re-acidification returned the pigment mostly into the ligroin phase as before.

Many more of the animals, brought in on the kelp, on July 13, 1936, were carefully removed and placed in glass tanks of running sea water. After twenty-four hours, during which interval no deaths occurred, and the anemones attached themselves to the walls of the container, about fifty animals were detached, rolled carefully and cleaned on filter paper to remove water and possible extraneous sources of pigment. The net weight of the mass of animals so treated was 13 grams. Upon dropping into pure methyl alcohol, much protein, probably of a mucous nature, was soon precipitated. The bodies of the anemones changed in color from the original orange or orange-pink to coral pink, while the alcohol took on an orange-yellow color. After storing overnight, under an inert atmosphere of illuminating gas, in a glass-stoppered bottle placed in an electric refrigerator, the methyl alcohol was decanted from the anemones, fresh 95% ethyl alcohol was added and

the tissues reduced to very fine particles by grinding for one and one-half hours in an Abbé ball mill. The mass was next extracted of all remaining pigment by washing with successive applications of alcohol, drawing the solvent off through a sintered glass filter. Finally the combined methyl- and ethyl-alcoholic extracts were treated in a closed system with alcoholic KOH in a hot water bath for two hours. The hydrolysate was stored under illuminating gas in the refrigerator overnight.

Dilution of the hydrolysate and a single extraction with ligroin yielded a very small amount of pigment to the latter solvent; subsequent washes yielded no more, all pigment remaining, as in the earlier experiment, in the aqueous alcoholic layer below, even after considerable dilution.

The trace of pigment extracted in ligroin from the diluted alkaline alcoholic hydrolysate was separated, carefully washed free of alcohol, freed of traces of water and passed through a micro-chromatographic column of dry $CaCO_3$; no pigment was adsorbed. The ligroin in the filtrate was evaporated, and the residual pigment taken up in CS_2, giving an orange-colored solution. This pigment was present in too small quantities to obtain good spectrophotometric measurements, but the facts that, after prolonged treatment with alcoholic KOH, it remained quantitatively epiphasic in the partition test between ligroin and 90% CH_3OH, and was unadsorbed by $CaCO_3$, marked it as a carotene. The main mass of pigment was now subjected to the same treatment as described in the first investigative procedure, and the earlier observations were duplicated; acidification freed the red carotenoid acid from combination as a potassium salt and rendered it extractable with ligroin; ligroin solutions of the pigment were orange-yellow; solid pigment residues were magenta-red in color, as were CS_2 solutions; partition tests of the free pigment between ligroin and 90% CH_3OH showed its solubility in both layers; alkalization of the system brought the pigment entirely back into the alcohol layer.

Chromatographic treatment of a ligroin solution of the free pigment upon a column of $CaCO_3$ showed a single slowly advancing rose-red zone. This procedure was repeated and gave an identical observation. The pigment was gradually washed through the column with added quantities of ligroin; when the solvent finally came through the column colorless, the filtered pigment solution was removed from the receiving flask and kept separately. Additional ligroin containing a little CH_3OH now desorbed a trace of pigment which had remained in the column. This trace was too small to attempt to classify it further. Its behavior suggests that it might have been a xanthophyll, present originally as an ester.

The absorption spectra of the free carotenoid acid and its ester, each in carbon disulphide, were determined separately by the direct visual method of matching light intensities through a Bausch and Lomb spectrophotometer. Figure 1 shows arbitrary units of relative intensity ($d = \log I_0/I$, where I_0

is the intensity of the light beam passing through the solvent alone, and I the intensity of the light beam passing through the solution of the pigment) plotted against λ, the wave-length in Å units. The lower curve of the ester and the upper curve of the free acid are not superposed, since there was no attempt to use identical concentrations (molecular weights being unknown) but merely to determine the region of the absorption maxima at any convenient concentration. It is seen that the pigment, whether free or esterified, shows a single broad band at 5000 Å with a high degree of symmetry in the slopes for a length of 500 Å on either side. (Compare asymmetry of astacene curve in pyridine, which has a considerably steeper slope between 5000 Å and 5500 Å than between 5000 Å and 4500 Å; illustrated in Kuhn and Lederer, 1933; Zechmeister, 1934; Lederer, 1935.)

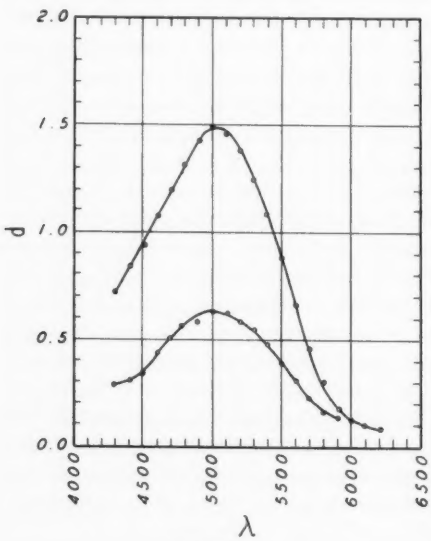

FIGURE 1

Absorption maximum of the chief carotenoid from *Epiactis prolifera;* upper curve, free acid; lower curve, natural ester. Ordinates: $d = \log I_0/I$. Abcissae: wave-length, λ, in Å.

Summary.—The small red-orange anemone, *Epiactis prolifera*, Verrill, which inhabits the ocean waters of the Pacific Coast, owes its striking color to considerable quantities of a red acidic carotenoid, present entirely in esterified form. The animal contains also a trace of one of the carotenes, and perhaps traces of an esterified xanthophyllic carotenoid. The character of the acidic carotenoid does not identify it with astacene, but its absorption spectrum renders the pigment more similar to certain other acidic carotenoids found in other species of anemones by Heilbron, *et al.*

The increasing number of known carotenoids peculiar to the animal kingdom emphasizes the question as to whether the biochemical differences in animal species are in the mode of action upon a single plant carotenoid, or in selective action of different animal species upon different plant carotenoids.

Acknowledgments.—We are grateful to Mr. M. J. Walsh, Manager of the Kelp Laboratories, San Diego, California, and to Mr L. Criswell, the skip-

per of Mr. Walsh's boat, for bringing to us the specimens of *Epiactis*. We are also indebted to various members of the local project of the Federal Works Progress Administration for aid in the preparation of the manuscript.

Contributions from the Scripps Institution of Oceanography. New Series No. 17.

Fox, D. L., "Further Studies of the Carotenoids of Two Pacific Marine Fishes, *Fundulus parvipinnis* and *Hypsypops rubicunda*, and of a Marine Annelid, *Thoracophelia* sp.," *Proc. Nat. Acad. Sci.*, **22**, 50–54 (1936).

Heilbron, I. M., Jackson, H., and Jones, R. N., "The Lipochromes of Sea Anemones, I. Carotenoid Pigments of *Actinia equina*, *Anemonia sulcata*, *Actinoloba dianthus* and *Tealia felina*," *Biochem. Jour.*, **29**, 1384–1388 (1935).

Johnson, M. E., and Snook, H. J., *Seashore Animals of the Pacific Coast*, Macmillan Co., New York (1927).

Karrer, P., and Loewe, L., "Constitution of Astacin," *Nature*, **133**, 936 (1934).

Karrer, P., and Solmssen, U., "Occurrence of Carotenoids in Some Marine Animals," *Helv. Chim. Acta*, **18**, 915–921 (1935).

Kuhn, R., and Lederer, E., "Taraxanthin, ein neues Xanthophyll mit vier Sauerstoffatomen," *Zeit. physiol. Chem.*, **200**, 108 (1931). (Cited by Zechmeister, 1934.)

Kuhn, R., and Lederer, E., "The Pigments of Lobster (*Astacus gammarus* L.) and Their Parent Substance, Astacin," *Ber.*, **66B**, 488–495 (1933).

Lederer, E., "Note sur les Lipochromes d'*Actinia Equina* L.," *Compt. rend. Soc. biol.*, **113**, 1391–1392 (1933).

Lederer, E., *Les Carotenoides des Animaux*, Paris, Hermann et Cie (1935).

Lederer, E., "Recherches sur les Carotenoides des Animaux Inférieurs et des Cryptogames." Thesis, University of Paris, Lons-le-Saunier Imprim erie et Lithographie Maurice Declume (1938).

Young, R. T., and Fox, D. L., "The Structure and Function of the Gut in Surf Perches (Embiotocidae) with Reference to Their Carotenoid Metabolism," *Biol. Bull.*, **71**, 217–237 (1936).

Zechmeister, L., *Carotinoide*, Berlin, Julius Springer (1934).

Zechmeister, L., "Die Carotinoide in tierischen Stoffwechsel," *Ergebn. der Physiol., biol. Chem., und exp. Pharmakol.*, **39**, 117–191 (1937).

THE STRUCTURE OF THE RING-GLAND (*CORPUS ALLATUM*) IN NORMAL AND LETHAL LARVAE OF *DROSOPHILA MELANOGASTER*

BY BERTA SCHARRER AND ERNST HADORN

DEPARTMENT OF ANATOMY, UNIVERSITY OF CHICAGO, AND DEPARTMENT OF ZOÖLOGY, UNIVERSITY OF ROCHESTER

Communicated April 29, 1938

In a recent paper (Hadorn, 1937a) it was shown that puparium formation in *Drosophila melanogaster* is brought about by a hormone. As the place of origin of this pupation hormone, an endocrine gland was described lying in the neighborhood of the brain. As long as no homology to other organs of that type in insects was found, this gland was called "ring-gland," as it is identical with the "Ring" of Weismann who in 1864 described it as a supporting structure for the dorsal blood vessel of *Calliphora vomitoria*.

Experiments with this ring-gland had the following results:

(1) Compared with normal larvae the puparium in the Drosophila mutation "lethal-giant" (*lgl-*) is formed later or not at all. This process can be accelerated by implantation of a mature normal ring-gland to *lgl*-larvae, but not by any other tissue (for instance brain) from genetically normal larvae.

(2) If ring-glands from normal larvae ready for pupation are implanted into younger normal larvae those will show a premature puparium formation. The degree of acceleration depends upon the time of injection and also upon the number of glands injected, three being more effective than one. The same accelerative effect can be obtained with mature normal ring-glands of *Drosophila melanogaster* in larvae of the more slowly developing *Drosophila hydei* and in lethal male crosses of the combination *Drosophila melanogaster* × *Drosophila simulans* (Hadorn, 1937b, Hadorn and Neel, 1938).

In the present paper the histological structure of the ring-gland was studied, the following questions being of particular interest:

(1) Is there histological evidence for the secretory activity of the ring-gland cells?

(2) Do ring-glands of old and young normal larvae (temporarily or permanently) show a different appearance indicating perhaps a functional cycle?

(3) Can normal ring-glands be differentiated from lethal ones?

(4) Is the ring-gland of Drosophila homologous with the corpora allata found in other insects?

1. *Material.*—As seen in table 1, normal and lethal larvae (*lgl-*) of *Drosophila melanogaster* were preserved at different ages in Bouin, Zenker-formol

or Champy. The material was embedded in celloidin, paraffin sections being used only as control material. The 5 μ serial sections (Rubaschkin method) were stained with Heidenhain's iron hematoxylin, Mallory-azan or van Gieson.

TABLE 1

AGE OF LARVA	GENETIC CONSTITUTION OF MATERIAL	NUMBER OF INDIVIDUALS INVESTIGATED
1 day	normal + *lgl*-larvae together	5
1 day + 19 hours	normal + *lgl*-larvae together	4
2 days + 22 hours	normal larvae	4
	lgl-larvae	4
3 days	normal + *lgl*-larvae together	3
3 days + 16 hours	normal larvae	4
	lgl-larvae	3
3 days + 22 hours	normal larvae	6
	lgl-larvae	2
4 days + 4 hours	normal larvae	4
	lgl-larvae	1
4 days + 16 hours	normal larvae	4
	lgl-larvae	1
4 days + 18 hours	normal larvae	4
	lgl-larvae	1
4 days + 22 hours	*lgl*-larvae	6
5 days + 5 hours	*lgl*-larvae	5
5 days + 20 hours	*lgl*-larvae	2
6 days	*lgl*-larvae	4

2. *Appearance of the Ring-Gland of a Normal Larva Ready for Pupation.*—As shown before in a diagram (Hadorn, 1937a, p. 480, Fig. 2), the ring-gland is located dorsally between the two hemispheres of the cerebral ganglion. Its longitudinal diameter is 200 μ, the largest transverse extension 130 μ and the depth 30 to 40 μ. As described in other Diptera by former investigators (Giacomini, 1900, Snodgrass, 1924), the "ring" is anchored in its position by tendinous structures. Tracheae penetrate and pass through the ring-gland; the cells surrounding them (imaginal bud cells, Burtt, 1937) can easily be differentiated from the gland cells. Two longitudinal tracheae arising from the cerebral hemispheres take their way forward and outward within the ventral part of the ring-gland. As

in Calliphora (Burtt, 1937), these two tracheal stems are connected by a transverse trachea. Significant is the fact that the ring encircles the aorta, the main part of the gland tissue being closely attached to the dorsal part of the vessel, whereas ventralward the ring becomes much thinner and ends more or less incompletely. The aorta itself can be traced from the heart to its termination in front of the ring-gland. The cells forming the thin wall of the vessel are thickened at those points where the oval-shaped nuclei are located. Sometimes one can observe in suitably stained sections that these cells contain a few delicate myofibrils showing a transverse striation; but only the part of the aorta posterior to the ring seems to be contractile. According to Giacomini (1900) the anterior portion of the dorsal vessel is incomplete in *Eristalis tenax*, consisting only of the upper half, whereas the lower lip ends immediately in front of the ring. The same is true in Drosophila; here this dorsal portion of the vessel is exceedingly thin, containing only comparatively large nuclei and ending in a membranous structure. Sometimes blood cells are found in the vessel especially in its frontal enlarged region.

To establish the relationship between ring-gland and stomatogastric nervous system is more difficult in Drosophila larvae than in other less tiny Diptera. One can trace the recurrent nerve passing forward and forming a small ganglion (evidently homologous to the median ganglion of Lowne, 1890–95, and Burtt, 1937), which is in contact with the anteroventral portion of the ring.

The cells forming the ring-gland are of two different types which can be differentiated by their size as well as by their position. The main part of the ring is built by large cell elements (about 15–40 μ in diameter) which are very characteristic. Their cytoplasm is deeply staining and shows a more or less granular structure. In some of the cells vacuoles of different size and irregular form can be found. The cell boundaries are easily distinguished in most of the preparations. The large nuclei of those elements are round or oval and have diameters from 10 to 17 μ; they contain one or two nucleoli (about 3 μ in diameter). The chromatin is arranged in more or less distinct chromosome-like bodies as, for instance, in Chironomus (Burtt, 1937). Thus these cells are similar in their general appearance to the elements of the salivary glands.

The cells of the smaller type form the central part of the ring-gland surrounding the anterior region of the ring-hole, and thus lying in the direct neighborhood of the dorsal vessel. Especially in its anterior part, this cell group shows a very intimate contact with the thin dorsal lip of the aorta. The area occupied by these smaller cells is about 60 μ long (from the ring-hole oralward in the anterior direction), 30 μ wide and 30–40 μ deep. In some preparations (azan) a very delicate connective tissue membrane can be observed surrounding this cell group as well as the whole ring. In a full

grown ring-gland of a mature larva one may count about 20 cells of that type. The smaller cells of the ring are in their general aspect similar to its larger elements. Their nuclei are round (5–6 μ in diameter) and have one nucleolus. As in the large ring cells, the nucleolar membrane is distinct. The cell diameters are from about 8 to 20 μ. The cytoplasm, being less vacuolated, stains somewhat deeper than that of the large cells. Nerve cells as described by Burtt (1937) in *Calliphora vomitoria* have not been found in the ring-gland of Drosophila. No direct histological evidence could be brought forward until now for a glandular activity of the ring-gland cells (i.e., presence of granules or colloid substances in the cytoplasm, etc.). But the general appearance of those cells (affinity of the cytoplasm to stains, vacuolization, large size of nuclei and their large amount of chromatin) do at least not speak against a secretory activity.

3. *Homology of Ring-Gland and Corpora Allata.*—Already in the first paper on Drosophila (Hadorn, 1937a) the question of a homology between ring-gland and corpora allata has been discussed. At the same time Burtt (1937) working with *Calliphora vomitoria* concluded that Weismann's ring represents a modified corpus allatum. He did not, however, prove this assumption by an embryological study, but he gave a comparative survey emphasizing the great variability in shape, size and position of the corpora allata not only within the insects but even within the Diptera. As yet the embryological development of the ring-gland in Drosophila has not been studied, this small species not being the most favorable object among the dipterous insects. But on the basis of a histological study of different postembryonic stages we came to the same result in Drosophila as Burtt in Calliphora. Among further morphological correspondences between ring-gland and corpora allata, we may mention their position in the body and relationship to other organs (especially to the dorsal vessel and to the tracheal system) and the histological properties of their elements (affinity to plasma stains, granular cytoplasm containing vacuoles, appearance of the nuclei, etc.).

There is one point more to be discussed concerning the two different cell types in the ring-gland of Drosophila. As mentioned above, the smaller cell elements have a well circumscribed position within the ring, being even separated from the large gland-cells by a fine membrane. Therefore the question arises whether this well defined central cell group forms a special part of the corpus allatum or must be separated from it as a different structure. It is very probable that the small ring-cells represent the corpora cardiaca (Pflugfelder, 1936–37, p. 47; esophageal ganglia, Burtt, 1937; pharyngeal ganglia of other investigators). Several reasons speak in favor of such an assumption:

(1) The name of corpora cardiaca was proposed by Pflugfelder for the "pharyngeal ganglia" of other authors, because of their intimate re-

lationship to the blood vessel system; in some insects (for instance in Nabis, Rhynchota) the corpora cardiaca represent even a modified and thickened wall of the heart or aorta, respectively. In Drosophila the close contact between the small ring-cells and the thin membrane of the dorsal vessel is obvious.

(2) Another reason for discarding the former name of pharyngeal or esophageal ganglia is that, at least in part of the insects, their purely ganglionic nature can no longer be maintained. Pflugfelder (1937a and b) showed that for instance in *Dixippus morosus* part of the cells of the corpora cardiaca contain (secretory or excretory?) osmio- and fuchsinophile substances. The small cells of the Drosophila ring have a glandular appearance rather than a nervous.

(3) The position of the corpora cardiaca in a number of insects (for instance in Syromastes, Rhynchota) is closely oralward of the corpora allata, with only a thin membrane separating these structures from each other. One may easily assume that in Drosophila the corpora allata on the basis of their special development have grown around their corpus cardiacum, thus building an even more intimate connection. The thin membrane found between the smaller and larger ring cells has already been mentioned.

(4) From other insects we know that the cell elements of the corpora cardiaca are smaller than those of the corpora allata.

(5) There has not been found as yet another organ in cyclorraphous insects which could be homologized with the corpora cardiaca.

In this connection we may ask what conditions we find in the other cyclorraphous insects. In the ring of Lucilia (we have studied only two specimens to date) we found also two cell types of different size and appearance, but they are not so separated from each other as in Drosophila. According to Burtt (1937, p. 21, Fig. 6) the same seems to be true for *Calliphora vomitoria*. Whether these smaller cells scattered in the ring-glands of Lucilia and Calliphora correspond to the group of small elements in the Drosophila ring cannot be decided at the present. In the same way the question of a homology between the small ring-gland cells and the corpora cardiaca will remain uncertain as long as we do not have a more detailed study of those organs in different groups of insects.

4. *Development of the Ring-Gland in Normal and Lethal Drosophila Larvae.*—In young larvae (age: 1 day, 1 day and 19 hours) the ring-gland elements (diameter of nuclei 3 μ) cannot yet be differentiated with certainty from the surrounding tissues, especially from the salivary glands. Normal larvae aged 2 days and 22 hours show already a typical ring. The ring-gland cells in this stage as well as in 3 day old larvae are to be distinguished from those of older ones by their size, the diameters of the nuclei being only from 6 to 8 μ. The smaller cells are already present in the center of the ring, but they do not differ so much in size from the large ones as later. In

three day old larvae the diameters of the small cell nuclei are from 4 to 5 μ. At the age of 3 days and 16 hours the ring-glands as well as their elements have reached their full size (Fig. 1) and now do not differ from the oldest normal stages described above.

Accordingly we may state that the growth of the ring-gland during the larval development of Drosophila takes place by an increase in the size of the cells, but not by a considerable increase in their number. That is why mitoses are to be found only exceedingly rarely in the ring-gland cells.

Also in the *lgl*-larvae the ring-gland has reached its final size at the age of 3 days and 16 hours but there is one striking difference in contrast to normal larvae. The ring-glands of *lgl*-larvae are considerably smaller than the normal. Even in 5–6 day old *lgl*-larvae (Fig. 2) the length of the ring is only about 120 μ, the width 70 μ and the depth 30 μ. As we may expect from what was said about the growth of the normal ring, the comparative smallness of the lethal ring-gland is due to the fact that the cells are smaller than normal. The nuclei of these large ring cells do not reach greater diameters than from 7 to 10 μ. In this connection it must be emphasized that an *lgl*-larva is as a whole not smaller than a normal one of the same age. Besides the size there have been observed no other marked differences between normal and lethal ring-gland cells, at least not with the technique used and on the basis of the present material. The small cell type in a lethal ring-gland does not differ in size from that in normal ones, neither are those cells less in number.

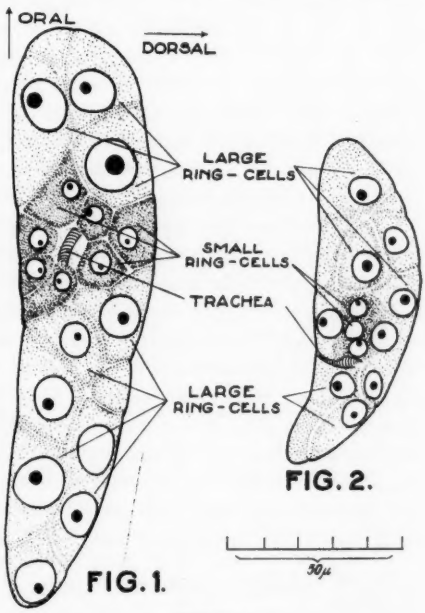

FIGURE 1

Paramedian sagittal section of ring-gland (corpus allatum) of a normal Drosophila larva (age 3 days and 16 hours). Camera lucida drawing.

FIGURE 2

Corresponding section of a lethal (*lgl*-) larva (age 5 days and 5 hours).

From the experiments of Hadorn (1937a) it was concluded that an *lgl*-

ring-gland does not furnish enough hormone to bring about puparium formation at the right time. The histological statements reported above may explain this fact. An *lgl*-ring-gland never grows to full size and apparently does not as a rule reach the hormone producing phase. It remains rather at a stage typical for a young normal larva of about 3 to $3^1/_2$ days. Unpublished transplantation experiments of Hadorn and Neel show that it is impossible to promote puparium formation with unripe glands even if they are of genetically normal constitution.

We are, however, not allowed to assume that an *lgl*-gland does not produce any hormone at all, since many of the lethal larvae later may form puparia. It is rather possible that an *lgl*-ring-gland, in spite of its underdeveloped histological structure, furnishes a small amount of hormone which reaches the necessary threshold only with great retardation.

A more detailed discussion of the literature will be given in another paper.

5. *Summary.*—The finer structure of the ring-gland (Weismann's "Ring") in *Drosophila melanogaster* is described. For the secretory activity of the ring-gland cells (pupation hormone) as suggested by Hadorn (1937) no histological evidence could be established. The ring-gland, or at least its greater part (large cells), is homologous to the corpora allata found in other insects. The question of a homology between the small ring-cells and the corpora cardiaca (Pflugfelder) is discussed. During the larval development of Drosophila the growth of the ring-gland (corpus allatum) takes place by an increase in the size of the cells. In the grown-up *lgl*-larvae the ring-gland as well as its cell elements is considerably smaller than normal. It is therefore concluded that the retardation in the puparium formation of the *lgl*-larvae is caused by an underdevelopment of their corpora allata.

This research was aided by a grant to the University of Chicago by the Rockefeller Foundation.

[1] Burtt, E. T., *Proc. Roy. Soc. London*, **B124**, 13–23 (1937).

[2] Giacomini, E., *Ann. Fac. Med.-Chir. Perugia*, **12**, 131–217 (1900; quoted from Snodgrass 1924).

[3] Hadorn, E., *Proc. Nat. Acad. Sci.*, **23**, 478–484 (1937a).

[4] Hadorn, E., *Naturwiss.*, **25**, 681–682 (1937b).

[5] Hadorn, E., and Neel, J., *Genetics*, **23**, 151 (1938).

[6] Lowne, B. T., *The Blow Fly*, London (1890–1895).

[7] Pflugfelder, O., *Zoologica* (ed. by R. Hesse) **34**, H. 93, 1–102 (1936–37).

[8] Pflugfelder, O., *Verhandl. Deutsch. Zool. Ges.*, Bremen (1937); *Zool. Anzeiger*, **10**, Suppl., 121 (1937a).

[9] Pflugfelder, O., *Z. wiss. Zool.*, **149**, 477–512 (1937b).

[10] Snodgrass, R. E., *Jour. Agric. Res.*, **28**, 1–36 (1924).

[11] Weismann, A., *Z. wiss. Zool.*, **14**, 187–336 (1864).

ON NORMAL DIVISION ALGEBRAS OF INDEX 5

BY RICHARD BRAUER

UNIVERSITY OF TORONTO

Communicated May 10, 1938

1. Let D be a normal division algebra of index $m = 5$ (i.e., of order 25) over a field F.[1] Very little is known concerning the structure of such a D, whereas the normal division algebras of smaller index have been completely discussed. In this paper, I will show that *there exists an extension field* $\mathfrak{F}^*$, *obtained by successively adjoining to* $\mathfrak{F}$ *the roots of two quadratic equations and one root of a cubic equation, such that the extended algebra* $D_{\mathfrak{F}^*}$ *over* $\mathfrak{F}^*$ *is cyclic.* This implies that D possesses soluble splitting fields, of degree at most 60. The question whether a given division algebra has soluble splitting fields resembles the question in the commutative case whether a given equation can be solved by means of radicals. Our method is analogous to the Tschirnhaus transformation of an equation. We shall have to solve four equations, of degrees 1, 2, 3, 4, respectively, for 25 parameters. We shall succeed in doing so after adjoining successively algebraic quantities of second and third degree to the ground field. The reason that the corresponding procedure does not work in the case of the solution of equations of the fifth degree is that there we have only five parameters instead of 25 parameters.

Our result for normal division algebras D of index 5 does not answer the question whether any such D itself is cyclic. However, it remains only to investigate algebras D with relatively simple splitting fields, and it may be hoped that this will greatly simplify the work on that problem.

The method given here can also be used for the investigation of normal division algebras of index $m = 3$ and $m = 4$; it yields a proof of the theorems of *J. L. M. Wedderburn*[2] and *A. A. Albert*[3] for these cases.

2. Let D be a normal division algebra of index m over a given field F. There exists always a separable splitting field $F(\vartheta)$ of degree m over F. Let $\vartheta_1 = \vartheta, \vartheta_2, \ldots, \vartheta_m$ be the conjugates of ϑ with regard to F. We denote the normal field $F(\vartheta_1, \vartheta_2, \ldots, \vartheta_m)$ by Ω, and its Galois group by $\mathfrak{G}$; and we set $\omega^G = \omega'$, if the number ω of Ω is carried into ω' by the element G of $\mathfrak{G}$. If $\vartheta_\mu^G = \vartheta_\nu$, we write $\nu = \mu G$. A conjugate triple system $c_{\alpha\beta\gamma}$ in Ω is a system of m^3 numbers $c_{\alpha\beta\gamma}$ $(\alpha, \beta, \gamma = 1, 2, \ldots, m)$ in Ω, such that the equation

$$c_{\alpha\beta\gamma}^G = c_{\alpha G, \beta G, \gamma G}$$

holds for every G in $\mathfrak{G}$ and for all α, β, γ. Similarly, conjugate double systems $l_{\alpha\beta}$ in Ω are systems of m^2 numbers of Ω for which $l_{\alpha\beta}^G = l_{\alpha G, \beta G}$ holds.

It was shown in a previous paper,[4] that to D there corresponds a system of m^3 numbers $c_{\alpha\beta\gamma}$ such that

$$(a) \qquad c_{\alpha\beta\gamma} \text{ is a conjugate triple system in } \Omega,$$
$$(b) \qquad c_{\alpha\beta\gamma}c_{\alpha\gamma\delta} = c_{\alpha\beta\delta}c_{\beta\gamma\delta} \quad (\alpha, \beta, \gamma, \delta = 1, 2, \ldots, m),$$
$$(c) \qquad c_{\alpha\beta\gamma} \neq 0 \text{ for all } \alpha, \beta, \gamma.$$

The system $c_{\alpha\beta\gamma}$ is called the factor system of D. The algebra D is then isomorphic to the algebra D_0 consisting of all matrices of degree m of the form

$$A = (l_{\kappa\lambda}c_{\kappa\lambda 1}) \qquad (\kappa, \lambda = 1, 2, \ldots, m) \qquad (1)$$

where $l_{\kappa\lambda}$ is any conjugate double system in Ω.

3. We form the characteristic polynomial of the matrix A

$$| xI - A | = x^m - h_1x^{m-1} + h_2x^{m-2} - \ldots + (-1)^m h_m$$

(I denotes the unit matrix of degree m). A simple computation gives

$$h_r = \sum_{\rho_1<\rho_2<\ldots<\rho_r}^{m} \sum l_{\rho_1\sigma_1} l_{\rho_2\sigma_2} \ldots l_{\rho_r\sigma_r} c_{\rho_1\sigma_1 1} c_{\rho_2\sigma_2 1} \ldots c_{\rho_r\sigma_r 1} \chi_{\sigma_1\sigma_2\ldots\sigma_r} \qquad (2)$$

where the inner sum is to be extended over all permutations $\sigma_1, \sigma_2, \ldots, \sigma_r$ of $\rho_1, \rho_2, \ldots, \rho_r$ and where $\chi(\sigma_1, \sigma_2, \ldots, \sigma_r)$ is $+1$ or -1 according as the permutation is even or odd.

We now put in (1)

$$\left.\begin{aligned} l_{\kappa\kappa} &= 0 \qquad (\kappa = 1, 2, \ldots, m), \\ l_{\kappa\lambda} &= (u_0 + u_1\vartheta_\kappa + \ldots + u_{m-1}\vartheta_\kappa^{m-1})^{-1} = \varphi(\vartheta_\kappa)^{-1}, \ (\kappa \neq \lambda) \end{aligned}\right\} \qquad (3)$$

where $u_0, u_1, \ldots, u_{m-1}$ are numbers in F which do not all vanish. Obviously, we obtain a conjugate double system. From (2) and (3) it follows that $h_r\varphi(\vartheta_1)\varphi(\vartheta_2)\ldots\varphi(\vartheta_m)$ is a homogeneous polynomial P_{m-r} of degree $m - r$ in $u_0, u_1, \ldots, u_{m-1}$:

$$h_r\varphi(\vartheta_1)\varphi(\vartheta_2)\ldots\varphi(\vartheta_m) = P_{m-r}(u_0, u_1, \ldots, u_{m-1}). \qquad (4)$$

We show that the coefficients of P_{m-r} lie in F. We apply an element G of $\mathfrak{G}$. Then h_r^G equals the corresponding coefficient in the characteristic equation of A^G, the image of A under G. We may, for a moment, consider $u_0, u_1, \ldots, u_{m-1}$ as indeterminates which are not changed by G. Put $\kappa G = \kappa'$, $\lambda G = \lambda'$, $1G = \zeta$. Then we have $A^G = (c_{\kappa'\lambda'\zeta}l_{\kappa'\lambda'})$, and after rearranging the rows and columns using the same permutation both times, we see that A^G is similar to

$$A_\zeta = (c_{\kappa\lambda\zeta}l_{\kappa\lambda}) \qquad (\kappa, \lambda = 1, 2, \ldots, m).$$

However, A and A_ζ are similar, since $A = QA_\zeta Q^{-1}$ with

$$Q = \begin{pmatrix} c_{1\zeta 1} & 0 & \cdots & 0 \\ 0 & c_{2\zeta 1} & \cdots & 0 \\ \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & \cdots & c_{m\zeta 1} \end{pmatrix}$$

as follows at once from the property (b) of the $c_{\alpha\beta\gamma}$. Consequently, A and A^G are similar and have, therefore, the same characteristic equation. This gives $h_r^G = h_r$. Since the factor of h_r in (4) is invariant under G, the same is true for the coefficients of $P_{m-r}(u_0, u_1, \ldots, u_{m-1})$. Hence these coefficients lie in F.[5]

The first equation (3) together with (2) shows that $h_1 = 0$. From (4) it follows that

$$P_{m-1}(u_0, u_1, \ldots, u_{m-1}) = 0 \qquad (5)$$

identically in $u_0, u_1, \ldots, u_{m-1}$. By the special choice of $l_{\kappa\lambda}$ in (3), we disposed of $m^2 - m$ parameters which appear in the general $l_{\kappa\lambda}$. We have, however, the advantage that the equation (5) of degree $m - 1$ is identically satisfied.

4. We now take $m = 5$. We choose u_0, u_1, u_2, u_3, u_4 from the equations

$$P_1(u_0, \ldots, u_4) = 0, \quad P_2(u_0, \ldots, u_4) = 0, \quad P_3(u_0, \ldots, u_4) = 0, \qquad (6)$$

which are homogeneous of degrees 1, 2 and 3. After eliminating one indeterminate by means of $P_1 = 0$, we may interpret the remaining u_ν as homogeneous coördinates in a 3-dimensional projective space. We have then to find a point on the intersection of a quadric and a cubic surface. We first determine a straight line s on the quadric. This may require the solution of two quadratic equations. After adjoining the roots to F we take the intersection of s with the cubic. Here we may have to solve a cubic equation. The adjunction of a root to F gives a soluble field F^*, of at most degree 12, and in F^* the equations (6) have a non-trivial solution.

We now replace F by F^*. The factor system of D remains the same. The element A of D, corresponding to the choice (3) of the $l_{\kappa\lambda}$ and (6) of $u_0, u_1, \ldots, u_4$, is different from 0. Its characteristic equation is

$$x^5 - h_5 = 0, \qquad (h_5 \text{ in } F^*).$$

Hence $F^*(\sqrt[5]{h_5})$ is a splitting field of D_{F^*}. This implies that D_{F^*} is a cyclic algebra over F^*.[6]

We had to solve two quadratic equations and one cubic equation in order to obtain F^*. If F has not the characteristic 2 or 3, we can obtain a field F^* such that D_{F^*} is cyclic, by successively adjoining three square roots and a cube root to F.

5. The same method can be used in the case of normal division alge-

bras of index $m = 3$. Here we have to consider only the linear equation $P_1 = 0$ in (6). Without any further adjunction, we obtain an element $A \neq 0$ in D, with a characteristic equation $x^3 - h_3 = 0$. This gives Wedderburn's result that all normal division algebras of index 3 are cyclic.

In the case $m = 4$, we again consider only the equation $P_1 = 0$ instead of (6). We obtain an element $A \neq 0$ in D, which has a characteristic equation of the form $x^4 - h_2x^2 - h_4 = 0$. The construction of such an element A forms the main part of Albert's proof of the theorem that every normal division algebra of index $m = 4$ possesses a Galois splitting field of degree 4 and can, therefore, be written as a crossed product, provided the characteristic of F is different from 2.

[1] For references to the theory of algebras, cf. M. Deuring's book: *Algebren* (*Ergebnisse der Mathematik*, Berlin, 1935).

[2] J. L. M. Wedderburn, *Trans. Amer. Math. Soc.*, **22**, 129 (1931).

[3] A. A. Albert, *Trans. Amer. Math. Soc.*, **31**, 253 (1929) and *Bull. Amer. Soc.*, **38**, 703 (1932).

[4] R. Brauer, *Math. Zeitschr.*, **30**, 79 (1929).

[5] This can also be seen from general considerations concerning simple algebras, without using the explicit form (1) of the elements.

[6] A. A. Albert, *Trans. Amer. Math. Soc.*, **36**, 885 (1934). For the case of a field F of characteristic 5, cf. A. A. Albert, *Trans. Amer. Soc.*, **39**, 183 (1936).

THE LIFT AND DRAG FUNCTIONS FOR AN ELASTIC FLUID IN TWO DIMENSIONAL IRROTATIONAL FLOW

By H. Bateman

Guggenheim Graduate School of Aeronautics,
California Institute of Technology

Communicated May 2, 1938

1. The lift function Y and the drag function X are defined by the equation

$$dX = (p + \rho u^2)dy - \rho uvdx = pdy + ud\psi = (p + \rho q^2)dy - \rho vd\phi,$$
$$dY = \rho uvdy - (p + \rho v^2)dx = vd\psi - pdx = \rho ud\phi - (p + \rho q^2)dx, \quad (1)$$

where u, v are the component velocities, p is the pressure and ρ is the density at the point x, y. The symbol q^2 is written for $u^2 + v^2$. Putting $x = r\cos\theta$, $y = r\sin\theta$, $v_r = u\cos\theta + v\sin\theta$, $v_\theta = v\cos\theta - u\sin\theta$ we have

$$dX = (p_\theta \sin\theta - \rho v_r v_\theta \cos\theta)dr + (p_r \cos\theta - \rho v_r v_\theta \sin\theta)rd\theta,$$
$$dY = (p_r \sin\theta + \rho v_r v_\theta \cos\theta)rd\theta - (p_\theta \cos\theta + \rho v_r v_\theta \sin\theta)dr, \quad (2)$$

where $p_r = p + \rho v_r^2$, $p_\theta = p + \rho v_\theta^2$.

Generalizing the analysis of Glauert and Lamb we write

$$\phi = V[r\cos\theta + A_0 + A_1\log r + A_2(\log r)^2 + \ldots + (1/r)\{B_0 + B_1\log r + B_2(\log r)^2 + \ldots\} + \ldots]$$

$$\psi = \rho_\infty V[r\sin\theta + C_0 + C_1\log r + C_2(\log r)^2 + \ldots + (1/r)\{D_0 + D_1\log r + D_2(\log r)^2 + \ldots\} + \ldots], \quad (3)$$

where the coefficients A_s, B_s, C_s, D_s are all functions of θ.

Differentiating these expressions with respect to r and θ we find

$$v_r = \frac{\partial\phi}{\partial r} = V[\cos\theta + (1/r)\{A_1 + 2A_2\log r + 3A_3(\log r)^2 + \ldots\} + (1/r)^2\{(B_1 - B_0) + (2B_2 - B_1)\log r + \ldots\} + \ldots],$$

$$v_\theta = (1/r)\frac{\partial\phi}{\partial\theta} = V[-\sin\theta + (1/r)\{A_0' + A_1'\log r + A_2'(\log r)^2 + \ldots\} + (1/r)^2\{B_0' + B_1'\log r + B_2'(\log r)^2 + \ldots\} + \ldots],$$

$$\rho v_r = (1/r)\frac{\partial\psi}{\partial\theta} = \rho_\infty V[\cos\theta + (1/r)\{C_0' + C_1'\log r + C_2'(\log r)^2 + \ldots\} + (1/r)^2\{D_0' + D_1'\log r + D_2'(\log r)^2 + \ldots\} + \ldots] \quad (4)$$

$$\rho v_\theta = -\frac{\partial\psi}{\partial r} = \rho_\infty V[-\sin\theta - (1/r)\{C_1 + 2C_2\log r + 3C_3(\log r)^2 + \ldots\} + (1/r)^2\{D_0 - D_1 + (D_1 - 2D_2)\log r + (D_2 - 3D_3)(\log r)^2 + \ldots\} + \ldots],$$

where primes denote differentiations with respect to θ.

We shall endeavor to express as many coefficients as possible in terms of A and C. To find the expressions we shall assume that

$$\rho = \rho_\infty[1 + R_0(1/r) + R_1(1/r)\log r + R_2(1/r)(\log r)^2 + \ldots + S_0(1/r)^2 + S_1(1/r)^2\log r + S_2(1/r)^2(\log r) + \ldots] \quad (5)$$

where the coefficients R_n, S_n are all functions of θ.

Using this expression for ρ to obtain series for ρv_r and ρv_θ from those for v_r and v_θ and then comparing the chief terms in the two sets of expansions we obtain a set of equations of which only four will be written down here as sufficient for our present needs but it may be mentioned that the other equations may be needed for a later calculation of the moment on a stationary obstacle.

$$\begin{aligned} R_0\cos\theta + A_1 &= C_0', & R_1\cos\theta + 2A_2 &= C_1', \\ -R_0\sin\theta + A_0' &= -C_1, & -R_1\sin\theta + A_1' &= -2C_2. \end{aligned} \quad (6)$$

To obtain another set of equations we shall suppose that the pressure and density are connected by the relation $p = f(\rho) - \rho f'(\rho)$ so that if c is the local velocity of sound, $c^2 = dp/d\rho = -\rho f''(\rho)$. We shall suppose,

moreover, that the arbitrary constant in $f'(\rho)$ is chosen so that the equation of Bernoulli is $^1/_2q^2 = f'(\rho)$. Then by Taylor's expansion, if $a^2 = (dp/d\rho)_\infty = -\rho_\infty f''(\rho_\infty)$, we have

$$^1/_2q^2 = {}^1/_2V^2 + (\rho - \rho_\infty)f''(\rho_\infty) + \ldots$$

Substituting the expression for $\rho - \rho_\infty$ and equating coefficients of $(1/r)$, $(1/r)\log r$, etc., we find that

$$V^2(A_1\cos\theta - A_0'\sin\theta) = -a^2R,\ V^2(2A_2\cos\theta - A_1'\sin\theta) = -a^2R_1 \tag{7}$$

the other equations being omitted though they may be needed in a calculation of the moment.

Writing $A_1 = A(a^2 - V^2\sin^2\theta)$, $C_1 = C(a^2 - V^2\sin^2\theta)$ we find that

$$A_0' = -V^2A\sin\theta\cos\theta - a^2C,\ C_0' = (a^2 - V^2)A - V^2C\sin\theta\cos\theta. \tag{8}$$

If the total circulation is $-K$, a finite quantity, we find that

$$K = -\int_0^{2\pi} v_\theta r d\theta = -V\int_0^{2\pi} A_0' d = V^3\int_0^{2\pi} A\sin\theta\cos\theta\, d\theta + a^2V\int_0^{2\pi} Cd\theta \tag{9}$$

and that $A_1, A_2, \ldots$ must be uniform functions of θ.

Furthermore, if there is no flow of matter to infinity on the whole C_0, C_1, $C_2, \ldots$ must be uniform functions of θ and so, in particular,

$$0 = \int_0^{2\pi} C_0' d\theta \therefore (a^2 - V^2)\int_0^{2\pi} Ad\theta = V^2\int_0^{2\pi} C\sin\theta\cos\theta d\theta. \tag{10}$$

We also find from equations (6) and (7) that

$$\begin{aligned}
R_0 &= -V^2(A\cos\theta + C\sin\theta),\\
R_1(a^2 - V^2\cos^2\theta) &= V^2A_1'\sin\theta - V^2C_1'\cos\theta,\\
2A_2(a^2 - V^2\cos^2\theta) &= a^2C_1' - V^2A_1'\sin\theta\cos\theta,\\
-2C_2(a^2 - V^2\cos^2\theta) &= A_1'(a^2 - V^2) + C_1'V^2\sin\theta\cos\theta,\\
p &= p_\infty + a^2(\rho - \rho_\infty) + \ldots\\
&= p_\infty - \rho_\infty V^2(A_1\cos\theta - A_0'\sin\theta)(1/r)\\
&\quad -\rho_\infty V^2(2A_2\cos\theta - A_2'\sin\theta)(1/r)\log r \text{ approximately.}
\end{aligned} \tag{11}$$

Combining the last equation with equations obtained by forming the squares and products of the series in equations (4) we find that to a first approximation

$$p_r = p_\infty + \rho_\infty V^2[\cos^2\theta + (1/r)(C_0' \cos\theta + A_0' \sin\theta) + (1/r)(\log r)(C_1' \cos\theta + A_1' \sin\theta)]$$

$$\rho v_r v_\theta = \rho_\infty V^2[-\sin\theta\cos\theta + (1/r)(A_0' \cos\theta - C_0' \sin\theta) + (1/r)(\log r)(A_1' \cos\theta - C_1' \sin\theta) \quad (12)$$

$$p_\theta = p_\infty + \rho_\infty V^2[\sin^2\theta + (1/r)(C_1 \sin\theta - A_1 \cos\theta) + (1/r)\log r\,(2C_2 \sin\theta - 2A_2 \cos\theta)].$$

Hence to a first approximation

$$dX = dr[(p_\infty + \rho_\infty V^2)\sin\theta + \rho_\infty V^2 C_1(1/r)] + d\theta\,[p_\infty r\cos\theta + \rho_\infty V^2(r\cos\theta + C_0' + C_1'\log r)]$$

$$X = (p_\infty + \rho_\infty V^2) r\sin\theta + \rho_\infty V^2(C_0 + C_1\log r); \quad (13)$$

thus X is a uniform function of θ.

$$-dY = dr[p_\infty\cos\theta + (1/r)(c_1\sin\theta\cos\theta - A_1\cos^2\theta + A_0'\sin\theta\cos\theta - C_0'\sin^2\theta) + (1/r)\log r\,(2C_2\sin\theta\cos\theta - 2A_2\cos^2\theta + A_1'\sin\theta\cos\theta - C_1'\sin^2\theta) + d\theta[p_\infty r\sin\theta + \rho_\infty V^2(A_0' + A_1'\log r + \ldots)] \quad (14)$$

$$Y = p_\infty r\cos\theta - \rho_\infty V^2\{A_0 + A_1\log r + \ldots\}.$$

Since A_1 is a uniform function of θ the total lift is $KV\rho_\infty$ as in the theorems of Lord Rayleigh, Kutta, Joukowsky and Glauert.

2. We now introduce the functions $\chi = ux + vy - \phi$, $\Omega = \rho uy - \rho vx - \psi$, which occur in the theory of the Legendre transformation of the partial differential equations satisfied by ϕ and ψ. Their approximate expressions are

$$\chi = r\frac{\partial\phi}{\partial r} - \phi = V[A_1 - A_0 + (2A_2 - A_1)\log r + \ldots + (1/r)\{B_1 - 2B_0 + (2B_2 - 2B_1)\log r + \ldots\} + \ldots] \quad (15)$$

$$\Omega = r\frac{\partial\psi}{\partial r} - \psi = \rho_\infty V[C_1 - C_0 + (2C_2 - C_1)\log r + \ldots + (1/r)\{D_1 - 2D_0 + (2D_2 - 2D_1)\log r + \ldots\} + \ldots].$$

We see that both χ and Ω are generally infinite for large values of r. When r is kept constant Ω is a uniform function of θ but χ is not; the change in χ as θ increases from 0 to 2π is for large values of r equal to K.

It should be remarked that if we write $u = q\cos\omega$, $v = q\sin\omega$, the functions χ and Ω are connected by the equations

$$\frac{\partial\Omega}{\partial\omega} = -q\rho\frac{\partial\chi}{\partial q}, \quad \frac{\partial\chi}{\partial\omega} = q\frac{\partial\Omega}{\partial(\rho q)}. \quad (16)$$

3. If, by means of equations (1) we express dX and dY in terms of $d\phi$ and $d\psi$ we obtain the equations

$$\begin{aligned} pd\phi &= vdX - udY, \\ (p + \rho q^2)d\psi &= \rho(udX + vdY), \\ \rho q^2 dX &= p\rho v d\phi + (p + \rho q^2)ud\psi, \\ \rho q^2 dY &= -p\rho u d\phi + (p + \rho q^2)vd\psi \end{aligned} \tag{17}$$

which may be compared with the equations

$$\begin{aligned} d\phi &= udx + vdy, \\ d\psi &= \rho udy - \rho vdx, \\ \rho q^2 dx &= \rho ud\phi - vd\psi, \\ \rho q^2 dy &= \rho vd\phi + ud\psi. \end{aligned} \tag{18}$$

Introducing a fictitious elastic fluid in the XY-plane and using large letters for the quantities relating to this fluid except for the density which we denote by σ, we have relations

$$P = -1/p, \quad P + \sigma Q^2 = -1(p + \rho q^2), \quad Q = q/p,$$

$$C^2 = \frac{dP}{d\sigma}, \qquad 1 - \frac{Q^2}{C^2} = \left(1 - \frac{q^2}{c^2}\right)\left(\frac{p}{p + \rho q^2}\right)^2 \tag{19}$$

$$\frac{dP}{dQ} = -\frac{q\rho}{p + \rho q^2} = -Q\sigma.$$

The last equation, which corresponds to $dp/dq = -q\rho$ shows that P, Q, σ are connected by the same relation as the pressure, velocity and density in an elastic fluid. We also have the relations

$$\begin{aligned} p(p + \rho q^2)dx &= \rho uvdX - (p + \rho u^2)dY, \\ p(p + \rho q^2)dy &= (p + \rho v^2)dX - \rho uvdY, \end{aligned} \tag{20}$$

$$P + \sigma U^2 = -\frac{p + \rho u^2}{p(p + \rho q^2)}, \quad P + \sigma V^2 = -\frac{p + \rho v^2}{p(p + \rho q^2)},$$

$$\sigma UV = -\frac{\rho uv}{p(p + \rho q^2)}$$

which show that there is a complete reciprocity between the real fluid and the fictitious fluid. Another correspondence between two elastic fluids may be obtained by introducing two arbitrary constants h, H and writing $x' = x + hY$, $y' = y - hX$, $X' = X + Hy$, $Y' = Y - Hx$. We then have the relations

$$vdX' - udY' = (p + H)d\phi, \quad \rho udX' + \rho vdY' = (p + \rho q^2 + H)d\psi$$

$$vdy' + udx' = (1 - hp)d\phi, \quad \rho udy - \rho vdx' = 1 - h(p + \rho q^2)d\psi. \quad (21)$$

Writing $\phi' = \phi$, $\psi' = (1 + hH)\psi$ and introducing primed quantities for a fictitious fluid in which ψ' is the stream function and ϕ' the velocity potential, we have the relations

$$p' = \frac{p + H}{1 - ph}, \quad u' = \frac{u}{1 - ph}, \quad v' = \frac{v}{1 - ph}, \quad q' = \frac{q}{1 - ph}$$

$$\rho' = \rho \frac{(1 - ph)(1 + Hh)}{1 - h(p + \rho q^2)}, \qquad p' + \rho' q^2 = \frac{p + \rho q^2 + H}{1 - h(p + \rho q^2)}$$

$$\frac{dp'}{dp} = \frac{1 + hH}{(1 - ph)^2}, \qquad \frac{dp'}{dq'} = -\rho' q'$$

$$1 - \frac{q'^2}{c'^2} = \left(1 - \frac{q^2}{c^2}\right) \frac{(1 - hp)^2}{[1 - h(p + \rho q^2)]^2}. \quad (22)$$

The last relation shows that $q'^2 > c'^2$ when $q^2 > c^2$. This is to be expected from the general behavior of characteristics in a point transformation.

H. Glauert, *Proc. Roy Soc. London*, **118**, 133, (1928).
H. Lamb, *Aeronautical Research Committee, R. M.*, 1156 (Ae 321) (1928).
E. Postolesi, *Le Alte Velocita in Aviazione*, 283, Rome (1936).

INFORMATION TO CONTRIBUTORS

THE PROCEEDINGS is the official organ of the NATIONAL ACADEMY OF SCIENCES and of the NATIONAL RESEARCH COUNCIL for the publication of brief accounts of important current researches of members of the ACADEMY and of the COUNCIL and of other American investigators. The PROCEEDINGS will aim especially to secure prompt publication of original announcements of discoveries and wide circulation of the results of American research among investigators in other countries and in all branches of science.

ARTICLES should be brief. The viewpoint should be comprehensive in giving the relation of the paper to previous publications of the author or of others and in exhibiting, where practicable, the significance of the work for other branches of science. Elaborate technical details of the work and long tables of data should be avoided, but authors should be precise in making clear the new results and should give some record of the methods and data upon which they are based.

MANUSCRIPTS should be prepared with a current number of the PROCEEDINGS as a model in matters of form, and should be typewritten in duplicate with double spacing, the author retaining one copy. Illustrations should be confined to text-figures of simple character, though more elaborate illustrations may be allowed in special instances to authors willing to pay for their preparation and insertion. Particular attention should be given to arranging tabular matter in a simple and concise manner.

REFERENCES to literature, numbered consecutively, will be placed at the end of the article and short footnotes should be avoided. It is suggested that references to periodicals be furnished in some detail and in general in accordance with the standard adopted for the Subject Catalogue of the International Catalogue of Scientific Literature, viz., name of author, with initials following (ordinarily omitting title of paper), abbreviated name of Journal, with place of publication, series (if any), volume, inclusive pages, year. For example: Montgomery, T. H., *J. Morph., Boston*, 22, 731–815 (1911); or, Wheeler, W. M., *Konigsburg, Schr. physik., Ges.*, 55, 1–142 (1914).

PAPERS by members of the ACADEMY may be sent to Edwin Bidwell Wilson, Managing Editor, Harvard School of Public Health, Boston 17, Mass. Papers by non-members of the ACADEMY or COUNCIL should be submitted through some member.

PROOF will not ordinarily be sent; if an author asks for proof, it will be sent with the understanding that charges for his corrections shall be billed to him. Authors are therefore requested to make final revisions on the typewritten manuscripts. The editors cannot undertake to do more than correct obvious minor errors.

REPRINTS should be ordered at the time of submission of manuscript. They will be furnished to authors at cost, approximately as follows:

SCHEDULE OF RATES FOR REPRINTS, CARRIAGE CHARGES EXTRA

No. copies	50	100	150	200	250
4 pp.	$2.10	$2.40	$2.70	$3.00	$3.30
8 pp.	4.20	4.80	5.40	6.00	6.60
Covers	1.60	2.05	2.50	2.95	3.40

CONTENTS

Page

CHEMISTRY.—EVIDENCE FOR A SECOND THIAMIN . . . . . *By William J. Robbins and Frederick Kavanagh* 229

ZOÖLOGY.—AN ASTACENE-LIKE CAROTENOID FROM A PACIFIC COAST ANEMONE, EPIACTIS PROLIFERA . . . . . *By Denis L. Fox and Chesney R. Moe* 230

GENETICS.—THE STRUCTURE OF THE RING-GLAND (CORPUS ALLATUM) IN NORMAL AND LETHAL LARVAE OF DROSOPHILA MELANOGASTER . . . . . *By Berta Scharrer and Ernst Hadorn* 236

MATHEMATICS.—ON NORMAL DIVISION ALGEBRAS OF INDEX 5 . . . . . *By Richard Brauer* 243

PHYSICS.—THE LIFT AND DRAG FUNCTIONS FOR AN ELASTIC FLUID IN TWO DIMENSIONAL IRROTATIONAL FLOW . . . . . *By H. Bateman* 246